SOLUTION OF THE DIRECT PROBLEM OF THE FLOW OF A TWO-PHASE MIXTURE OF A GAS AND FOREIGN SOLID OR LIQUID PARTICLES IN A LAVAL NOZZLE

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Within the framework of a two-liquid (two-velocity and two-temperature) model of a continuous medium, the article considers the flow of a mixture of a gas and foreign particles in the subsonic, transonic, and supersonic parts of a Laval nozzle. In the case of a thin layer of pure gas near the wall, the problem is solved in two stages. First, the method of establishment is used to calculate the core of the flow, where the gas with the particles is flowing; under these circumstances, the parameters in the layer of pure gas are determined approximately; then simplified equations (of the type of the equations of the boundary layer) are used to find the distribution of the parameters in the zone of pure gas, and the flow in the core of the stream is refined. Examples of the calculation are given. Use of the method developed permitted establishing some of the special characteristics of the flow of a mixture of gas with particles in a Laval nozzle in the case of Stokes flow around the foreign particles.

The method of establishment was used in [1, 2] to solve the direct problem of the flow of a mixture of a gas with particles in a Laval nozzle, in a two-dimensional statement. However, due to the lag of the particles, a layer of pure gas is formed at the wall. This layer can be rather thin but, with any arbitrarily small thickness, in the case of a finite relative mass flow rate of the particles (the mass flow rate of the particles to the mass flow rate of the mixture), the parameters of the gas in the layer change by a finite amount. This latter circumstance complicates considerably the use of the method of establishment in the case of a small thickness of the layer near the wall, since to achieve a satisfactory degree of accuracy in a layer of pure gas would require a rather small division, which would lead to a considerable increase in the calculation time of the problem.

In [3] the problem of the flow of a mixture of a gas with particles in a Laval nozzle was solved using the method of perturbations. It was postulated that the coefficients φ^f and φ^q, determining the interaction between the particles and the gas, are great. The solution was found in the form of expansions in terms of the small parameters $\varepsilon_1 = 1/\varphi^f$ and $\varepsilon_2 = 1/\varphi^q$. Simplified equations were obtained, describing the flow in a layer of pure gas near the wall. It was noted that, since the small parameter appears in the equations only through the thickness of the layer, which, in the case considered in the cited article, is proportional to ε_1, then exactly the same relationships will be valid with any arbitrary value of ε_1 for a layer of gas whose thickness is small in comparison with the characteristic dimension of the nozzle.

In accordance with what has been said above, in the present work the solution of the direct problem of the flow of a mixture of gas with particles in a Laval nozzle, in the case of a sufficiently thin layer near the wall, is carried out in two stages. First, the method of establishment is used to calculate the core of the flow, where the gas with the particles is flowing. The problem is solved with a division which is coarse for the layer near the wall, but sufficient from the point of view of accuracy in the core. Then simplified equations based on the distributions of the parameters of the gas along the line of separation, obtained by the method of establishment, are used to find the flow in the layer near the wall; this is followed by a refinement of the flow in the core. The use of the method developed in the work has made it possible to es-

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tablish and study some of the special characteristics of the flow of a mixture of a gas and particles in a Laval nozzle, with a Stokes law.

Let us consider the flow of a mixture of a gas and foreign particles in an axisymmetric Laval nozzle. We locate the origin of a cylindrical system of coordinates in the minimal cross section of the nozzle, we direct the x axis along the axis of the flow toward the side of the motion, and the y axis is perpendicular to the x axis. It is assumed that there are no coagulation, phase transformations, external forces, or heat sources, and that the volume of the particles is negligibly small in comparison with the volume of the gas. We assume that the flow under consideration can be described within the framework of a model of a two-liquid continuous medium. The equations of the flow of such a medium are given, for example, in [2].

Within the framework of the above model, the interaction between the gas and the particles is due to the force **f** with which the gas acts on the particles, and to the heat flux q from the gas to the particles; here by f and q there are understood quantities relating to one particle, referred to its mass. For f and q the following expressions are adopted: $\mathbf{f} = \varphi^f(\mathbf{W} - \mathbf{W}_s)$; $q = \varphi^q(T - T_s)$, where W and T are the vector of the velocity and the temperature of the gas, and W_s and T_s are the analogous values for the particles. In what follows, the coefficients φ^f and φ^q will be assumed constant, which corresponds to the conditions of Stokes flow around each particle. We note that the latter assumption with respect to the conditions of flow around the particles is fundamental from the point of view of the method used.

We consider a perfect gas with constant heat capacity and adiabatic index $\varkappa$. The specific internal energy of the particles e_s is a linear function of their temperature T_s, i.e., $e_s = \delta T_s$, where δ = const is the specific heat capacity of the particles.

All the quantities in the relationships given and in what follows will be dimensionless. Let L, W_*, ρ_* be characteristic quantities with the dimensionalities of length, velocity, and density, and let R be the dimensional value of the gas constant. Then reduction to dimensionless form is achieved by referring the spatial variables to L, the velocities to W_*, the densities to ρ_*, the pressure to $\rho_* W_*^2$, the enthalpy and the internal energy to W_*^2, the temperature to W_*^2/R, the heat capacity of the particles to R, the force f to W_*^2/L, and the heat flux q to W_*^3/L. As L there is taken the radius of the minimal cross section of the nozzle, and ρ_* and W_* are taken as the critical density and velocity of the mixture with equilibrium flow, i.e., flow without lag of the particles with respect to the velocity or the temperature.

The solution of the steady-state problem is obtained during the process of the establishment of the pressure. The boundary conditions are taken to coincide with the boundary conditions of the corresponding steady-state problem. It is assumed that the nozzle is joined smoothly to a semiinfinite cylindrical tube. Then, with $x \to -\infty$, there is flow without dynamic (with respect to the velocity) or thermal (with respect to the temperature) lag of the particles, with vertical components of the velocities of the gas and the particles equal to zero. The distributions of the total enthalpy and entropy of the mixture, and the ratio of the density of the particles to the density of the gas are assumed constant over the cross section. It is well known (see, for example, [3]) that the equilibrium flow of a mixture of a gas and particles is equivalent to the flow of a gas with a density $\rho_\Sigma = \rho + \rho_s$ and an effective adiabatic index $\varkappa_e$, which is defined using the following relationships:

$$\varkappa_e = \frac{\beta}{\beta - 1} \quad \left(\beta = \frac{\varkappa}{\varkappa - 1} + \frac{1 - m}{m}\delta\right).$$

Here ρ, ρ_s, ρ_Σ are the densities of the gas, the particles, and the mixture, respectively, and $m = \rho/(\rho + \rho_s)$ is a given constant, equal to the relative mass flow rate of the gas with $x \to -\infty$. In carrying out the calculations, the boundary conditions were carried to a sufficiently distant cross section $x = x_0$ in the cylindrical part of the channel.

At the wall of the channel and at the axis the condition of impermeability for the gas is satisfied. For the particles, such a boundary condition is not set. However, it is postulated that there is no reflection of particles from the wall and no intersection of the flow lines of the particles in the field of the flow. The satisfaction of the latter condition can be verified after solution of the problem. When this condition breaks down, a description of the flow within the framework of a two-liquid medium becomes impossible.

The outlet cross section of the nozzle is taken so far in the expanding part that there the condition $u > a$ is satisfied, where u is the projection of the velocity of the gas **W** on the x axis; $a = \sqrt{\varkappa p/\rho}$ is the speed of sound in the gas; therefore, in this cross section no kind of additional boundary conditions must be set.

The steady-state field of the flow is obtained during the process of establishment with respect to the time. The special characteristics of the difference scheme used with establishment are set forth in [2].

The distributions of the parameters along the line of separation between the layer of pure gas and the core of the flow, obtained as a result of establishment, using simplified equations proposed in [3], are used to find the flow in the layer near the wall.

The flow in the layer of pure gas is considered in the curvilinear system of coordinates τn, connected with the wall of the nozzle; the τ and n axes are directed along the tangent and along a normal to the wall on the gas side, respectively. It must be noted that this approach permits calculation of the layer near the wall over cross sections normal to the wall. The distribution of the parameters in the layer of pure gas, with an accuracy up to 0 (ε), where ε is a small parameter, equal to the ratio of the thickness of the layer to the characteristic dimension of the problem in the cross section τ=const, is found from the following system of equations:

$$i(p,\rho)+U^2/2=H(\psi);\ S(p,\rho)=S(\psi);\ \frac{\partial p}{\partial n}=\rho U^2 K,\ \ d\psi=-cy^{\nu}\rho U dn,$$

where U is the projection of the vector of the velocity of the gas on the τ axis; ψ is the stream function; c is a constant, which is so selected that, with $\psi=0$ at the axis of symmetry, the value of the stream function at the wall ψ_w will be equal to unity; S is the entropy; K is the curvature of the wall, and the functions $H(\psi)$ and $S(\psi)$ are determined from the values of the total enthalpy and entropy of the gas at the line of separation. As an independent variable, it is convenient to consider ψ. The parameters are determined successively, from the line of separation where $\psi=\psi_d$ (the subscript d relates to the line of separation) to the point where $\psi=\psi_w$; here ψ_w is found from the value of the stream function at the point of the descent of the line from the wall. Under these circumstances, the line $\psi=\psi_w$ will not coincide with the given wall of the nozzle, which is connected with the errors admitted into the calculations, and with the effect of the nonuniformity of the flow in the layer of pure gas.

The contour of the nozzle is corrected taking account of the thickness of the displacement, and the flow is calculated again using the method of establishment. In the following stage, the parameters in the layer near the wall are determined. Calculations have shown that, even after a single recalculation of the contour, the line $\psi=\psi_w$ is located considerably closer to the upper wall of the given nozzle than with the original determination of the parameters in the layer near the wall. The deviation of the line $\psi=\psi_w$ from the starting contour of the nozzle with the conditions under consideration was approximately 1%. The process of successive approximations described above can be continued.

The basic calculations were made for an axisymmetric nozzle, whose contour is given in the following manner. The constricting and expanding parts were formed by segments of straight lines, with angles of inclination to the x axis of 30 and 15°, respectively. The rectilinear sections were smoothly connected together by the arc of a circle of unit radius (all the dimensions were referred to the ordinate of the minimal cross section of the nozzle). The constricting part of the nozzle joins the arc of a circle of radius r = 2, going over smoothly to a cylindrical section of the same radius. The constants $\varkappa$ and δ were taken equal to 1.4 and 0.7, respectively.

In Figs. 1 and 2, where the scale along the y axis is twice as large as along the x axis, the solid curves illustrate lines of constant Mach numbers of the gas for the cases m = 1/2 and 1/4, respectively; here $\varphi^f=2$ and $\varphi^q=4$. The Mach number was calculated from the speed of sound in the gas $M=W/a$. The dashed line shows the line of separation. It must be noted that the shift of the sonic line M = 1 downstream from the minimal cross section depends strongly on the relative mass flow rate of the particles and, at

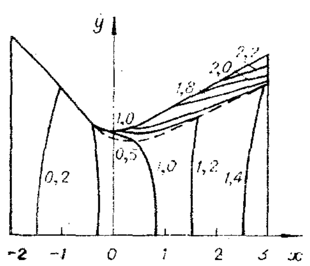

Fig. 1

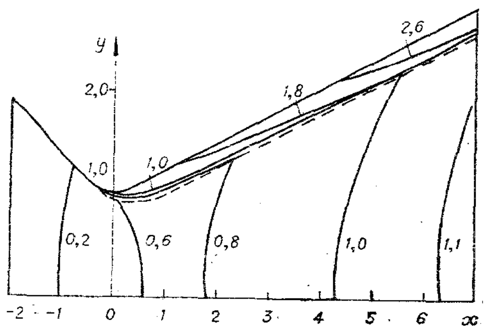

Fig. 2

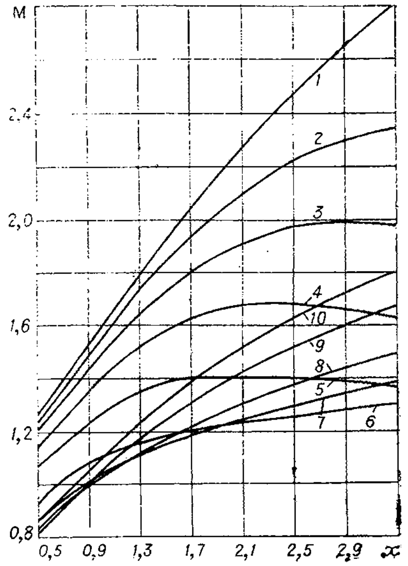

Fig. 3

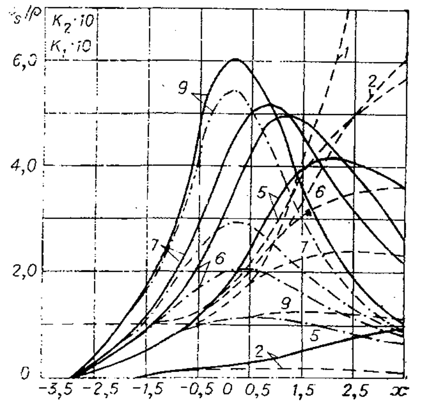

Fig. 4

the axis, amounts to 0.85 and 4.29 radii of the throat of the nozzle, with m = 1/2 and 1/4, respectively. In the layer near the wall, there is intense acceleration of the flow; the flow there is essentially nonuniform.

During the course of the calculations, the parameter φ^f was varied; here the value of φ^q was always equated to the doubled value of φ^f. Figure 3 shows the distribution of M for the gas along the axis of the nozzle in its expanding part with m = 1/2. Curves 1-10 in Figs. 3 and 4 correspond to values of the interaction coefficient of 0; 0.02; 0.05; 0.1; 0.2; 0.5; 1.0; 2.0; 8.0, and to equilibrium flow without velocity or temperature lag of the particles (the latter corresponds to $\varphi^f = \infty$). It must be noted that M in the outlet cross section of the nozzle is always less than the value corresponding to frozen flow, where there is no interaction between the phases (however, it can be either greater or less than the value corresponding to equilibrium flow). In some cases, the presence of particles leads to a situation in which, with a rise in the value of x, the Mach number at the axis of the nozzle starts to decrease. The latter can be explained by the predominance of the braking action of the particles over the acceleration connected with the expansion of the nozzle.

Figure 4 shows the change in some of the quantities which determine the force action of the particles on the gas, for different interaction coefficients. A decrease in φ^f leads to an increase in the lag of the particles with respect to the velocity. At the same time, as can be seen from Fig. 4, the value of $K_1 = \varphi^f(1 - W_s/W)$, whose distribution along the axis of the nozzle is shown by the dashed-dot curve, increases monotonically with a rise in the value of φ^f, for all values of x. However, the force action of the particles on the gas is determined not only by the lag of the particles with respect to the velocity, but also by the ratio of the densities of the particles and the gas. The change in the value of ρ_s/ρ along the axis of the nozzle for different values of φ^f is shown by the dashed curves. It must be noted that, with a transition from equilibrium flow conditions to frozen conditions, there is a rise in the ratio of the densities of the particles and the gas over almost the whole length of the nozzle. The latter leads to a nonmonotonic change, with respect to φ^f, in the value of $K_2 = \varphi^f(1 - W_s/W)\rho_s/\rho$, taking account both of the contribution of the dynamic lag of the particles, and of the ratio of the densities in the force action of the particles on the gas. The distributions of the value of K_2 along the x axis for different values of φ^f are shown by the solid curves. The above-noted nonmonotonicity may be the reason for the previously discussed special characteristic in the behavior of M for the gas at the axis of the nozzle with a change in the value of φ^f. Article [4], which gives a review on one-dimensional two-phase flow in nozzles, also contains data which bear witness to a nonmonotonic change in the velocity of the gas with an increase in the interaction coefficient.

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INVESTIGATION OF TRANSONIC UNSTEADY-STATE FLOW IN THE PRESENCE OF PHASE TRANSFORMATIONS

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INTRODUCTION

Condensation of supersaturated vapor in a transonic flow can lead to an unsteady-state character of the flow. This is due to the evolution of the latent heat of condensation, to the formation of a shock wave, and to its interaction with the zone of evaporation. This phenomenon was first noted in [1, 2], in which it is shown that the character of the motion of the shock wave depends on the parameters in the initial cross section, the relative moisture content, and the contour of the nozzle. In [3] there were measured considerable pulsations of the parameters of the flow (with a frequency of 500-1000 Hz), arising with the flow of moist air and pure water vapor in air. In [4] an approximate law of similarity was introduced for the dimensionless frequency of an unsteady-state flow. In communications [5, 6] the phenomenon under consideration was studied by the method of the inversion of the action; [7, 8] give the results of theoretical calculations and an experimentally confirmed diagram, making it possible to determine the boundaries of the region of instability of the flow. It has been found recently that the frequency of the pulsations of the pressure and the density in a flow with the condensation of moist air can attain 6000 Hz. In the present work, a modification of the method of Godunov [10] is used to obtain a numerical solution of a system of equations describing an unsteady-state quasi-one-dimensional flow with spontaneous condensation in the transonic part of a Laval nozzle. Calculations of nonequilibrium unsteady-state flows in nozzles by the method of establishment have also been made previously, for example, in [11, 12] (mixed flow in nozzles), [13] (flow taking account of vibrational relaxation and nonequilibrium chemical reactions), and [14] (two-phase flow in a nozzle, with disagreement of the phases with respect to velocities and temperatures). The specific characteristic of the present problem consists in the fact that, during the process of establishment with steady-state initial and boundary conditions, the limiting state is not steady-state; however, a known periodicity is observed.

1. Let us consider the unsteady-state quasi-one-dimensional flow of supersaturated vapor in a Laval nozzle, without taking account of viscosity, thermal conductivity, or radiation. We assume that the velocities of the phases are identical, and that the condensation is spontaneous. The dependence of the area of the transverse cross section of the nozzle on the coordinate x, varying along the axis, is given by the function F(x); here x = 0 corresponds to the minimal cross section of the nozzle. Let p be the pressure, ρ the density of the mixture, u the velocity, and t the time; the parameters of the condensing phase have the superscript zero. The basic equations of the conservation of mass, momentum, and energy can be written in the form

$$\frac{\partial}{\partial t}(\rho F)+\frac{\partial}{\partial x}(\rho u F)=0;$$

$$\frac{\partial}{\partial t}(\rho u F)+\frac{\partial}{\partial x}[(p+\rho u^2)F]=p\frac{\partial F}{\partial x};$$

$$\frac{\partial}{\partial t}\left[\rho F\left(h-\frac{p}{\rho}+\frac{u^2}{2}\right)\right]+\frac{\partial}{\partial x}\left[\rho u F\left[h+\frac{u^2}{2}\right)\right]=0. \quad (1.1)$$

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